SOME ASPECTS OF THE MACHINE COMPUTATION OF HEAT-EXCHANGER TUBE WALL TEMPERATURES

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The author describes a computer method of calculating the temperature at any point on a heat-exchanger tube wall.

The mathematically simplified, yet clumsy methods of calculating the wall temperatures of heat-exchanger tubes [1, 2] recommended for engineering purposes make it necessary to solve this problem on a computer.

Since there are no heat sources in the tube wall and the temperature field is stationary, to determine the field it is necessary to solve the Laplace equation $\nabla^2 t^* = 0$ or in polar coordinates

$$\frac{1}{r}\frac{\partial t^*}{\partial r} + \frac{\partial^2 t^*}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 t^*}{\partial \psi^2} = 0 \qquad (1)$$

with the boundary conditions

$$r = r_{\text{in}}, \quad q_{\text{in}} = \lambda\left(\frac{\partial t^*}{\partial r}\right)_{r=r_{\text{in}}} = \alpha_2 t^*_{\text{in}} \qquad (1a)$$

when the temperatures are determined at points on the inner wall of the tube, and

$$r = r_{\text{ex}}, \quad q_{\text{ex}} = \lambda\left(\frac{\partial t^*}{\partial r}\right)_{r=r_{\text{ex}}} \qquad (1b)$$

when the temperatures are determined at points on the external wall of the tube.

Here, q is some known function of the angle ψ measured around the periphery from the front point of the tube.

The general solution of Eq. (1) is found in the form [3]:

$$t^* = A_0 \ln\frac{r}{r_{\text{in}}} + B_0 + \sum_{n=1}^{\infty}\left[\left(A_n r^n + a_n\frac{1}{r^n}\right)\cos n\psi + \left(B_n r^n + b_n\frac{1}{r^n}\right)\sin n\psi\right]. \qquad (2)$$

Expanding $q(\psi)$ in a Fourier series, we obtain

$$q(\psi) = q_{\text{m}} + \frac{1}{\pi}\sum_{n=1}^{\infty}(\alpha_n \cos n\psi + \beta_n \sin n\psi),$$

where

$$q_{\text{m}} = \frac{1}{2\pi}\int_0^{2\pi} q(\psi)\,d\psi; \quad \alpha_n = \int_0^{2\pi} q(\psi)\cos n\psi\,d\psi;$$

$$\beta_n = \int_0^{2\pi} q(\psi)\sin n\psi\,d\psi.$$

The function $q(\psi)$, which characterizes the heat load on the tube, can be calculated at any point; accordingly, henceforth in referring to this function we will have in mind a tabulated function. Since it is symmetrical (even), solution (2) takes the form

$$t^* = A_0 \ln\frac{r}{r_{\text{in}}} + B_0 + \sum_{n=1}^{\infty}\left(A_n r^n + a_n\frac{1}{r^n}\right)\cos n\psi. \qquad (2a)$$

For boundary conditions (1a) and (1b) it is easy to show that in (2a)

$$A_0 = \frac{r_{\text{ex}} q_{\text{m}}}{\lambda}; \qquad B_0 = \frac{r_{\text{ex}}}{r_{\text{in}}}\frac{q_{\text{m}}}{\alpha_2};$$

$$A_n = \frac{r_{\text{ex}}}{\pi n\lambda} \times \left(r_{\text{ex}}\alpha_2 + n\lambda\right) \times \left[(r_{\text{in}}\alpha_2 + n\lambda)\left(\frac{r_{\text{ex}}}{r_{\text{in}}}\right)^n + (r_{\text{in}}\alpha_2 - n\lambda)\left(\frac{r_{\text{in}}}{r_{\text{ex}}}\right)^n\right]^{-1} \times \left(\frac{1}{r^n_{\text{in}}}\right)\alpha_n; \qquad (3)$$

$$a_n = -\frac{r_{\text{ex}}}{\pi n\lambda} \times \left(r_{\text{in}}\alpha_2 - n\lambda\right) \times \left[(r_{\text{in}}\alpha_2 + n\lambda)\left(\frac{r_{\text{ex}}}{r_{\text{in}}}\right)^n + (r_{\text{in}}\alpha_2 - n\lambda)\left(\frac{r_{\text{in}}}{r_{\text{ex}}}\right)^n\right]^{-1} \times r_{\text{in}}\alpha_n;$$

$$\alpha_n = 2\int_0^{\pi} q(\psi)\cos n\psi\,d\psi;$$

$$q_{\text{m}} = \frac{1}{\pi}\int_0^{\pi} q(\psi)\,d\psi. \qquad (4)$$

Table 1

Values of the Series Sum Σ_n as a Function of n

n	Σ_n	n	Σ_n	n	Σ_n
1	5.681359	5	6.560868	9	6.561883
2	6.665318	6	6.561757	10	6.561883
3	6.574674	7	6.561975	11	6.561883
4	6.557871	8	6.561912	12	6.561883

Thus, in order to solve the problem it is necessary to construct a table of values of $q(\psi)$ or, in other words, the heat load rosette. To determine the heat load at any point on the periphery of the tube we use the equation [1]

$$q_0 = (\vartheta - t) \Big/ \left[\beta \left(\frac{s}{\lambda} \frac{2}{\beta + 1} + \frac{1}{\alpha_2} \right) + \right.$$

$$\left. + \frac{1}{\alpha_c + \alpha_r} + \varepsilon \right],$$

where the heat transfer coefficients α_r and α_c vary from point to point. The coefficient of heat transfer from the wall to the medium α_2 is assumed to be independent of ψ (for a single-phase medium inside the tube).

The heat transfer coefficients for different points on the periphery of the tube are determined as follows.

In accordance with the recommendations given in [1], the convective heat transfer coefficient α_c is determined from the equation

$$\alpha_c = k_t\, \alpha_{c.m}.$$

The radiative heat transfer coefficient α_r is found from the equation

$$\alpha_r = \alpha_{r.v}\, \varphi_v + \alpha^*_{r.v}\, \varphi^*_v + (1 - \varphi_v - \varphi^*_v)\, \alpha_{r.bt}.$$

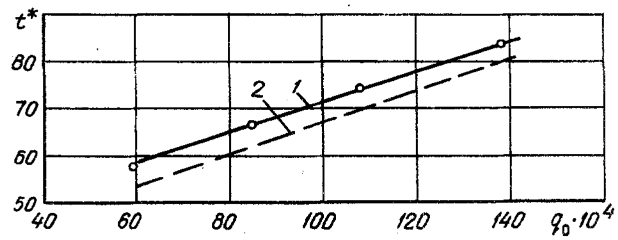
The calculation of the view factors φ_v and φ^*_v, which usually involves clumsy geometrical constructions unsuitable for computer operations, is replaced by computation from the corresponding analytic expressions.

After the heat load rosette has been constructed, to find t* from (2a) it is necessary to evaluate the integrals in (3) and (4).

Whereas the evaluation of $\int_0^\pi q(\psi)\, d\psi$ does not present special difficulties and can be carried out by ordinary numerical methods (for example, Simpson's rule), the evaluation of $\int_0^\pi q(\psi) \cos n\psi\, d\psi$ requires special consideration.

The fact is that evaluation of the integral over the entire interval of a function strongly oscillating about zero, for example, by Simpson's rule (even with automatic step selection) or by Gauss's method, leads to a serious loss of accuracy. This shortcoming can be eliminated as follows.

The entire interval of integration is divided into segments with respect to zeros of the function $\cos n\psi$,



Relation between t*, °C, and the heat load q_0, W/m^2, at $\psi = 0$: 1) experimental data; 2) calculated curve.

i.e., the interval $[0, \pi]$ is divided into $(n + 1)$ segments $[0, \psi_0]$, $[\psi_0, \psi_1]$, $[\psi_1, \psi_2]$, ... $[\psi_{n-1}, \pi]$, where

$$\psi_i = \frac{2i + 1}{2} \pi,$$

and the sum of the partial integrals

$$\sum_{i=1}^{n} J_i = \int_0^\pi q(\psi) \cos n\psi\, d\psi$$

is determined, where

$$J_i = \int_{\psi_{i-1}}^{\psi_i} q(\psi) \cos n\psi\, d\psi.$$

In this case it is necessary to know the values of the integrand at a large number of points on the interval $[0, \pi]$.

In order to shorten the calculations it is desirable to use on each segment $[\psi_{i-1}, \psi_i]$ a parabolic approximation of the tabulated function, selecting the tabulated values so that the parabola describes the behavior of the function $q(\psi)$ precisely on the given segment. Then

$$q(\psi) = a\psi^2 + b\psi + c,$$

and

$$\int_{\psi_{i-1}}^{\psi_i} (a\psi^2 + b\psi + c) \cos n\psi\, d\psi$$

is evaluated directly.

Calculations on a "Ural-2" computer showed that the accuracy of determination of the unknown tempera-

Table 2

Comparison of the Results of Temperature Calculations

ψ	Values of t					
	from [2]		calc.		deviation	
	t_{in}	t_{ex}	t_{in}	t_{ex}	t_{in}	t_{ex}
0	143	72	145	75	+2	+3
60	148	84	150	87	+2	+3
120	156	106	156	106	0	0
180	157	109	157	107	0	—2

tures is perfectly satisfactory. The accuracy of the solution depends on two basic factors: the convergence of the series from (2a) and correct construction of the table of values of the functions $q(\psi)$.

In order to illustrate the convergence of the series we present values of the sum $\sum_n$ for n varying from 1 to 12 in a typical calculation (Table 1).

Further increases in n have no effect on the sum. As a rule, in evaluating the series sum it is sufficient to take the first ten terms.

The second factor affecting the accuracy of the solution is best analyzed by comparing the calculated and experimental temperatures.

It is clear from Fig. 1 that the discrepancy between the experimental* and calculated values does not exceed 4° C.

It is worthwhile comparing the solutions obtained by the method proposed with solutions found by other methods.

Thus, in [2] Schneider gives an example of the determination of the wall temperatures of a heat exchanger by partitioning a hollow cylinder with a logarithmic net and solving a clumsy system of equations (pp. 207–209). The results are compared in Table 2.

On a computer the time required to determine the wall temperatures at three points is about four minutes (at $r = r_{in}$; $r = r_{ex}$ and $r = (r_{ex} + r_{in})/2$).

The short machine times and satisfactory accuracy make it possible to use this program to investigate numerous variants.

*The experimental data were obtained by Kh. Dimerchan on the steam superheater of the high-pressure boiler at the LenGES-1 hydroelectric power station.

NOTATION

$t^* = t_w - t_m$ is the temperature difference between some point on the tube wall and the medium flowing through the tube; r is the variable radius indicating the position of the point at which the temperature is determined; r_{ex}, r_{in} are the external and inside radii of tube; λ is the thermal conductivity of metal; α_2, α_c, α_r are the coefficients of heat transfer from wall to medium, by convection and by radiation, respectively; s is the thickness of tube wall assumed given in the first approximation and then subject to refinement as a function of the temperature found; $\beta = d_{ex}/(d_{ex} - 2s)$ (here d_{ex} is the external diameter); ϑ is the temperature of the heating medium; ε is the coefficient of contamination of external surface of tubes; t is the temperature of the heated medium; k_t is the tube variation factor; $\alpha_{c.m}$ is the convective heat transfer coefficient averaged over the periphery; $\alpha_{r.v}$, $\alpha^*_{r.v}$ are the radiative heat transfer coefficients of volumes before and after the heat exchanger; φ_v and φ^*_v are the tube view factors with respect to volumes before and after the heat exchanger

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